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Transverse structure functions in high-Reynolds-number turbulence

Brindesh Dhruva, Yoshiyuki Tsuji, and Katepalli R. Sreenivasan
Mason Laboratory, Yale University, New Haven, Connecticut 06520-8286

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Transverse structure functions are obtained at high Reynolds numbers in atmospheric turbulence (Taylor microscale Reynolds numbers between 10 000 and 15 000). These measurements confirm that their scaling exponents are different from those for longitudinal structure functions. Implications of this conclusion are discussed briefly. [S1063-651X(97)50511-3]

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Anomalous scaling in turbulence has been studied traditionally in terms of the so-called longitudinal structure functions (LSF's), which are moments of velocity increments $\Delta u_r \equiv u(x+r)-u(x)$, where u is the velocity component in a certain direction x and the separation distance r is measured also in the same direction. For most flows, experimental convenience necessitates that the direction x be that of the mean flow. Several attempts [1–10] have been made recently to obtain the so-called transverse structure functions (TSF's), which are moments of velocity increments for which the separation distance is transverse to the direction of the velocity component considered. A few of these measurements (e.g., Refs. [1,2,6]) suggest (or imply) that the scaling exponents for TSF are equal, to within experimental uncertainties, to those for LSF. If the two sets of exponents are indeed equal, the hierarchy of models built up on the basis of LSF (see, e.g., Ref. [11]) remains essentially intact. On the other hand, there exist measurements [3–5,8–10] purporting to show that the transverse exponents of order greater than 2 are measurably smaller than the longitudinal exponents. If true, this observation calls for additional complexity in small-scale phenomenology—and might even suggest the absence of strict scaling in the problem.

To make a convincing case that high-order TSF exponents are smaller than those of LSF, it must first be shown that the inertial-range scales are isotropic. A minimum condition for local isotropy to exist is that the second-order exponents in the inertial range should be equal for LSF and TSF. It is known (e.g., Ref. [12], Fig. 5) that this requires, in shear flows, a Taylor microscale Reynolds number of the order of 1000 and higher. All the results cited above have been obtained at modest Reynolds numbers. Some of them have been made in shear flows. One might therefore wonder if the observed differences between the two sets of exponents are due perhaps to the lack of isotropy in the inertial range. Further, the scaling range at moderate Reynolds number is modest at best.

In this context, we have made a series of measurements in atmospheric turbulence at Taylor microscale Reynolds numbers ranging between 10 000 and 15 000. These Reynolds numbers are comparable to the highest ever used for studies of small-scale turbulence (e.g., [13,14]). Here, we examine the velocity data solely to address the following issue: Are there genuine differences between the longitudinal and transverse exponents? As already remarked, this question is important for the theory of small-scale turbulence.

The velocity data were acquired by means of single-wire and $\times$-wire probes mounted at a height of about 35 m above the ground on a meteorological tower at the Brookhaven National Laboratory. The hot wires were about 0.7 mm in length and 5 μm in diameter. They were calibrated just prior to being mounted on the tower, and operated on DISA 55M01 constant-temperature anemometers. The frequency response of the hot wires was typically good up to 20 kHz. The voltages from the anemometers were low-pass filtered and digitized. The low-pass cutoff was never more than half the sampling frequency f_s. The voltages were converted to velocities in a standard way through the calibration procedure. The mean wind velocities, roughly constant over the duration of a given data set, ranged between 5 and 10 ms^{-1} in the experiment series. The usual procedure of surrogating time for space (''Taylor's hypothesis'') was used to obtain the dissipation rate $\langle \varepsilon \rangle$ and estimate the Kolmogorov scale η. The latter varied between 0.44 and 0.64 mm among the various data sets and was comparable to the active wire length. The real-time duration of data records was typically of the order 2500 sec. Table I lists the relevant data for the data records analyzed here: $\overline{U}$ and u' are the mean and root-mean-square velocities, respectively, and f_s is the sampling frequency;

$$\langle \varepsilon \rangle = 15\nu \langle (\partial u \partial x)^2 \rangle ,$$

$$\eta = (\nu^3 / \langle \varepsilon \rangle)^{1/4} ,$$

TABLE I. The basic parameters for the data sets used in this work.

$\overline{U}$ ms^{-1}	u' ms^{-1}	$\langle \varepsilon \rangle$ $\mathrm{m}^2\ \mathrm{s}^{-3}$	η mm	λ cm	R_λ	f_s, Hz per channel	Number of samples
7.6	1.36	3.2×10^{-2}	0.57	11.4	10 340	5000	10^7
4.8	1.45	2.0×10^{-2}	0.64	15.4	14 860	2000	5×10^6
5.2	1.80	0.92×10^{-2}	0.44	8.9	10 680	10 000	4×10^7

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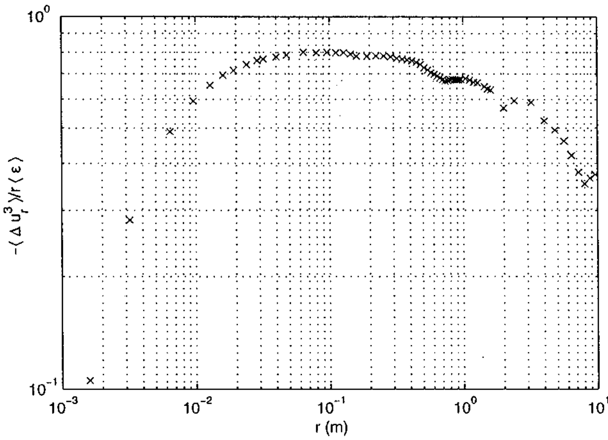

FIG. 1. Normalized third-order structure function, $S_3 \equiv \langle \Delta u_r^3 \rangle / r \langle \varepsilon \rangle$, where $\varepsilon = 15\nu\langle(\partial u/\partial x)^2\rangle$, plotted against the separation distance r. A scaling range of more than a decade is likely. The magnitude of S_3 is not far from 0.8 in the region where it is roughly flat, as expected from Kolmogorov's four-fifths law; it is both interesting and nontrivial that the law appears to hold quantitatively even in inhomogeneous turbulence. First and second sets of data from Table I have been combined.

$$\lambda = \left(\frac{15\nu u'^2}{\langle \varepsilon \rangle} \right)^{1/2},$$

$$R_\lambda = u'\lambda/\nu.$$

Figure 1 shows the normalized third-order LSF, $S_3 \equiv -\langle \Delta u_r^3 \rangle / r \langle \varepsilon \rangle$, plotted against the separation distance r. If the turbulence were homogeneous, one expects a sizeable region where S_3 is a constant equal to 4/5 [15]. There exists no foolproof demonstration that the Kolmogorov's four-fifths law holds for inhomogeneous turbulence (e.g., see [16,17]). Atmospheric turbulence is strongly inhomogeneous. Yet, the expectation just cited is roughly satisfied. The flat region in Fig. 1 can thus be considered the inertial range. It is, however, difficult to choose from the figure an unambiguously scaling part. This choice is critical if small differences in scaling exponents are being sought. We have sidestepped the issue here by computing the ratios of LSF to TSF as functions of r. If the longitudinal and transverse exponents are the same, the ratios must be flat over some range. We shall consider $n = 2$, 4, and 6; data convergence is poorer for $n > 6$. It is also useful [18] to examine moment orders below unity. For this purpose, one should take absolute values of velocity differences in the above expression. We thus consider, in general, the ratios of generalized structure functions,

$$R_n = \langle |\Delta u_r|^n \rangle / \langle |\Delta v_r|^n \rangle,$$

where v is the turbulent velocity normal to the ground.

Figure 2 shows the ratios R_2, R_4, and R_6. As already noted, isotropy considerations demand that R_2 should have zero slope in the inertial range. This is indeed very nearly so. Cross-spectral data (not presented here) confirm that the anisotropy is negligible in this range of scales. If one fits a power law to R_2 in Fig. 2, one obtains an index of about 0.02. This is quite close to zero. On the other hand, the ratios

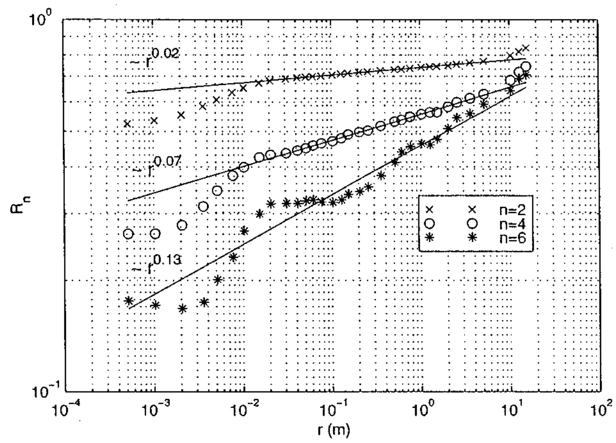

FIG. 2. Ratios R_2, R_4, and R_6 plotted against r. As expected for locally isotropic turbulence, the ratio R_2 is essentially independent of r, while the fourth- and sixth-order ratios become increasingly stronger functions of r. This confirms that higher-order TSF's scale with smaller exponents than LSF's of corresponding order. Third set of data from Table I has been used.

R_4 and R_6 show stronger departures from being flat in the same region. Least square fits yield $R_4 \sim r^{0.07}$ and $R_6 \sim r^{0.13}$. Even though the scatter in R_6 is large, the slope can be obtained relatively unambiguously. If the structure functions scale like power laws, as is believed to be the case (e.g., Ref. [16]), the indices in R_4 and R_6 are equal to the differences between the longitudinal and transverse exponents. It appears clear that the latter are larger than the former. The results are very close to those obtained from low-Reynolds-number simulations [9].

One can also obtain the two sets of exponents directly by using the extended self-similarity method [19]; as examples, the fourth-order LSF and TSF are plotted in Fig. 3 against $\langle |\Delta u_r|^3 \rangle$. The scaling region is marked. For this flow, the power-law part does not extend to the dissipation region, consistent with Ref. [20]. It is clear that the TSF has a smaller slope than the LSF. The difference in the slope agrees well with that obtained from Fig. 2 directly. Table II

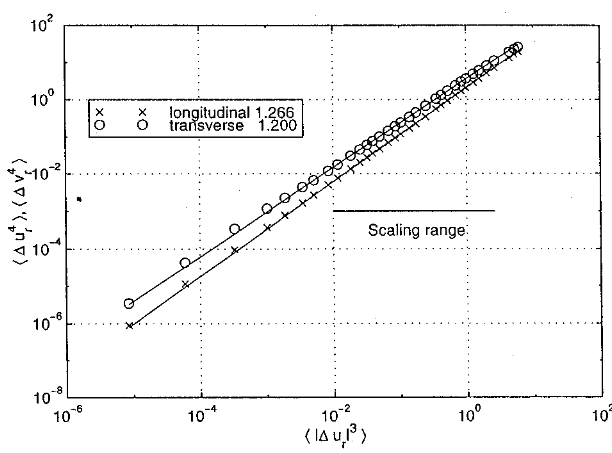

FIG. 3. ESS plots for the fourth-order LSF and TSF. Marked on the figure are the scaling region and the slopes obtained from least square fits to data in that range. Third set of data from Table I has been used.

TABLE II. Scaling exponents for generalized structure functions determined from the ESS method. For exponents with moment orders 4, 5, and 6, the error bars are, respectively, ± 0.022, ± 0.031, and ± 0.05, and are comparable for longitudinal and transverse exponents. For lower moment orders, the errors are much smaller but harder to quantify.

Moment order	Longitudinal exponent	Transverse exponent
0.2	0.076	0.075
0.4	0.150	0.148
0.6	0.224	0.220
0.8	0.295	0.290
1.0	0.366	0.359
2.0	0.700	0.680
3.0	1	0.960
4.0	1.266	1.200
5.0	1.493	1.402
6.0	1.692	1.567

lists the exponents for the *generalized* LSF and TSF obtained by the ESS method. For reference, it might be noted that $\langle|\Delta u_r|^3\rangle \sim \langle \Delta u_r^3\rangle^{1.05}$ for the present measurements.

In summary, it appears that the scaling exponents for TSF are measurably smaller than those for LSF. The second-order structure functions should scale exactly alike if local isotropy prevails in the inertial range. In our data, they are slightly different (Fig. 2). One might thus wonder if this residual anisotropy gets magnified in higher-order moments, leading to the present conclusion. However, this lingering anisotropy is no more than that observed in the DNS data of formally isotropic turbulence [9]. We are inclined to think that it is almost impossible to obtain textbook isotropy in the inertial range, and that the level of anisotropy present in our data is benign. Subject to this provision, we support the previous conclusion drawn from studies using numerical data (e.g., Ref. [9]) that the transverse exponents are smaller than the longitudinal ones. The numerical data correspond to modest Reynolds numbers, but have the advantage of not needing Taylor's hypothesis. While we do employ the hypothesis, its effects are believed to be small because only ratios of LSF to TSF are considered.

If the two sets of exponents are indeed different, as appears likely, it would mean that one needs a richer small-scale phenomenology than is usually employed (e.g., Ref. [11]). An attempt in this direction has been made by Chen *et al.* [9]. Alternatively, it has been suggested [21] that structure functions are not the fundamental objects of interest. The suggestion of Ref. [21] is to use, instead, irreducible representation of the rotation group. At the level of the fourth order, the latter reduces to a linear combination of the LSF, the TSF, and the mixed structure function, $\langle \Delta u_r^2 \Delta v_r^2\rangle$ [22]. We have tested the scaling of these linear combinations. Purely on empirical grounds based on the quality of power-law plots, it does not appear that there is any advantage to preferring them over the traditional structure functions. A more detailed discussion of this important issue is beyond the scope of this article and will be published elsewhere.

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